Mark Scheme (Results)

Summer 2014

Pearson Edexcel International A Level in Further Pure Mathematics F2 (WFM02/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $\sqsubset$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q \neq 0$, leading to x = …

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme		Marks
1.(a)	$\dfrac{1}{2(r+1)(r+2)}-\dfrac{1}{2(r+2)(r+3)}$ $=\dfrac{r+3-(r+1)}{2(r+1)(r+2)(r+3)}$	Attempt common denominator	M1
	$=\dfrac{2}{2(r+1)(r+2)(r+3)}$ $=\dfrac{1}{(r+1)(r+2)(r+3)}$	Correct proof	A1
			(2)
(a) Way 2	$\dfrac{1}{(r+1)(r+2)(r+3)}=\dfrac{1}{r+2}\left(\dfrac{1}{(r+1)(r+3)}\right)=\dfrac{1}{r+2}\left(\dfrac{1}{2(r+1)}-\dfrac{1}{2(r+3)}\right)$		M1
	$\dfrac{1}{2(r+1)(r+2)}-\dfrac{1}{2(r+2)(r+3)}$		A1
	M1: Factor of $\dfrac{1}{r+2}$ and attempt partial fractions A1: Correct proof		
	Other methods**: Complete** method scores M1 All work correct inc final answer reached A1		
(b)	$\displaystyle\sum_{r=1}^{n}\frac{1}{(r+1)(r+2)(r+3)}=$ $=\dfrac{1}{12}-\dfrac{1}{24}+........$ $+\dfrac{1}{2(n+1)(n+2)}-\dfrac{1}{2(n+2)(n+3)}$	Attempt at least the first pair and the last pair of terms as shown. Must start at 1 and end at n	M1
	$=\dfrac{1}{12}-\dfrac{1}{2(n+2)(n+3)}$	Identifies that the first and last terms do not cancel.	M1
	$=\dfrac{n^2+5n+6-6}{12(n+2)(n+3)}$	Correctly combined fractions	A1
	$=\dfrac{n^2+5n}{12(n+2)(n+3)}$ or $\dfrac{n(n+5)}{12(n+2)(n+3)}$ *	Allow either form isw attempts to multiply out the denominatot	A1
			(4)
			Total 6

Question Number	Scheme		Marks
2.	$\dfrac{6}{x-3} \le x+2$		
	Way 1		
	$\dfrac{6}{x-3} \le x+2 \Rightarrow x+2-\dfrac{6}{x-3} \ge 0$		
	$x+2-\dfrac{6}{x-3} \ge 0 \Rightarrow \dfrac{(x+3)(x-4)}{x-3} \ge 0$	Attempt to combine fractions and factorise the numerator	M1
	$x=-3, \quad x=4$	Correct critical values	A1, A1
	$x \ge 4$	Follow through their 4	A1ft
	$x=3$	Identifies 3 as a critical value	B1
	$-3 \le x < 3$	M1: Attempt inside region A1: Correct inequality	M1A1
			(7)
	Way 2		
	$6(x-3) \le (x+2)(x-3)^2$ $\Rightarrow (x-3)(4-x)(x+3) \ge 0$	Multiplies both sides by $(x-3)^2$ and attempt to factorise	M1
	$x=-3, \quad x=4$	Correct critical values	A1, A1
	$x \ge 4$	Follow through their 4	A1ft
	$x=3$	Identifies 3 as a critical value	B1
	$-3 \le x < 3$	M1: Attempt inside region A1: Correct inequality	M1A1
			(7)
	Way 3		
	$x-3>0 \Rightarrow 6 \le (x+2)(x-3)$ $\Rightarrow (x-4)(x+3) \ge 0$	Multiplies both sides by $(x-3)$ and attempt to factorise Must state $x-3>0$	M1
	$x=4$	Correct critical values	A1
	$x \ge 4$	Follow through their 4	A1ft
	$x=3$	Identifies 3 as a critical value	B1
	$(x-3<0) \Rightarrow 6 \ge (x+2)(x-3)$ $x=-3$	Correct critical value	A1
	$(x+2)(x-3) \le 6 \Rightarrow (x-4)(x+3) \le 0$ $\Rightarrow -3 \le x < 3$	M1: Attempt inside region A1: Correct inequality	M1A1
			(7)

Question Number	Scheme		Marks
3.			
	$r^5 = \sqrt{16^2 + (16\sqrt{3})^2} = 32 \Rightarrow r = 32^{\frac{1}{5}} \left(= 2\right)$	Correct value for r	B1
	$\arg(16 - 16i\sqrt{3}) = \frac{5\pi}{3}$	Allow $\frac{5\pi}{3}$ or $-\frac{\pi}{3}$	B1
	$5\theta = \frac{11\pi}{3}, \frac{17\pi}{3}, \frac{23\pi}{3}, \frac{29\pi}{3}$	$\left(\frac{5\pi}{3}\right) + 2n\pi, n = 1, 2, 3, 4$ At least 2 values which must be positive. May be implied by correct final answers.	M1
	$z = \underline{2e^{\frac{\pi}{3}i}}, \underline{\underline{2e^{\frac{11\pi}{15}i}, 2e^{\frac{17\pi}{15}i}, 2e^{\frac{23\pi}{15}i}, 2e^{\frac{29\pi}{15}i}}}$	2 or $32^{\frac{1}{5}}$, $e^{\frac{5\pi}{15}i}$ or $e^{\frac{\pi}{3}i}$	B1 A1(all 4 values)
			(5)
			Total 5

Question Number	Scheme		Marks
4	$w=\dfrac{z}{z+3}$		
	$w=\dfrac{z}{z+3}\Rightarrow z=\dfrac{3w}{1-w}$	M1: Attempt to make z the subject A1: Correct expression for z	M1A1
	$\lvert z\rvert=2\Rightarrow\left\lvert\dfrac{3w}{1-w}\right\rvert=2$ $\lvert 3w\rvert=2\lvert 1-w\rvert$ $9\left(u^2+v^2\right)=4\left(u-1\right)^2+4v^2$	M1: Uses $\lvert z\rvert=2$ to obtain an equation in u and v Pythagoras must be used correctly. No i seen A1: Any correct equation in u and v Isw attempts to simplify	M1A1
	$5u^2+5v^2+8u-4=0$		
	$\left(u+\dfrac{4}{5}\right)^2+v^2-\dfrac{16}{25}-\dfrac{4}{5}=0$	Rearrange to a suitable form for a circle and attempt centre and/or radius. -16/25 (from completing the square) may be omitted. May be implied by centre and radius correct for their previous equation	M1
	Centre $\left(-\dfrac{4}{5},0\right)$	oe	A1
	Radius $\dfrac{6}{5}$	oe	A1
			(7)
			Total 7

Question Number	Scheme		Marks
5	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 2x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 0$		
(a)	$y''' - 2y' - 2xy'' + 2y'(=0)\left(y''' = 2xy''\right)$	M1: Attempt to differentiate including use of the product rule on $2x\frac{\mathrm{d}y}{\mathrm{d}x}$ Equation may have been re-written as $y'' = ...$ before differentiating A1: Correct differentiation	M1A1
	$y'''' - 2y'' - 2xy''' - 2y'' + 2y''(=0)$	M1: Second use of product rule. Dependent on first M1. A1: Correct differentiation NB A simpler form is obtained if $y''' - 2xy'' = 0$ is used.	dM1A1
	$y'''' = 2xy''' + 2y''$ $= 2x\left(2xy''\right) + 2y'' = \left(4x^2 + 2\right)y''$	Cao and cso	A1
			(5)
(b)	$y_0'' = -2,\ \ y_0''' = 0\ \ \ y_0^{iv} = -4$	B1: $y_0'' = -2$ M1: Attempts y_0''' and y_0^{iv} A1: All correct and obtained from correct expressions	B1, M1A1
	$(y=)1 + 3x - x^2 - \frac{x^4}{6}$	M1: Correct use of Maclaurin series A1: Fully correct expansion.	M1A1
			(5)
(c)	$(y=)1 + 3(-0.2) - (-0.2)^2 - \frac{(-0.2)^4}{6}$	Use of the correct Maclaurin series and substitution of $x = -0.2$	M1
	$(y=)\ 0.3597$	Allow awrt	A1
			(2)
			Total 12

Question Number	Scheme		Marks
6.	$x\dfrac{dy}{dx}+(1-2x)y=x$		
	$\dfrac{dy}{dx}+\dfrac{(1-2x)}{x}y(=1)$	Divides by x - may be implied by subsequent working	M1
	Integrating factor $I=e^{\int\frac{1-2x}{x}dx}$	Correct attempt at I, including an attempt at the integration. ln must be seen if not fully correct.	dM1
	$=e^{\ln(x)-2x}$	Correct expression	A1
	$=xe^{-2x}$	No errors in working allowed	A1
	$xye^{-2x}=\int xe^{-2x}\,dx$	Multiply through by their IF and integrate LHS. Can be given if y x their IF is seen	M1
	$=-\frac{1}{2}xe^{-2x}+\frac{1}{2}\int e^{-2x}\,dx$	M1: Correct application by parts ie differentiate x and attempt to integrate e^{-2x} A1: Correct expression	M1A1
	$=-\frac{1}{2}xe^{-2x}-\frac{1}{4}e^{-2x}\ (+c)$	A1: Complete the integration to a correct result. Constant not required.	A1
	$y=\dfrac{ce^{2x}}{x}-\dfrac{1}{4x}-\dfrac{1}{2}$	Oe Must have $y=\ldots$ Must include a constant. ft their previous line	A1ft
			(9)
			Total 9

Question Number	Scheme		Marks
7	$P: \lvert z+1 \rvert = \lvert 2z-1 \rvert$		
	$z = x+\mathrm{i}y \Rightarrow \lvert x+\mathrm{i}y+1 \rvert = \lvert 2(x+\mathrm{i}y)-1 \rvert$		
	$(x+1)^2 + y^2 = (2x-1)^2 + (2y)^2$	M1: Correct use of Pythagoras A1: Any correct equation	M1A1
	$Q: \lvert w \rvert = \lvert w-1+\mathrm{i} \rvert$		
	$w = x+\mathrm{i}y \Rightarrow \lvert x+\mathrm{i}y \rvert = \lvert x+\mathrm{i}y-1+\mathrm{i} \rvert$		
	$x^2 + y^2 = (x-1)^2 + (y+1)^2 \; (\Rightarrow y = x-1)$	M1: Correct use of Pythagoras. Allow with *u* and *v* instead of *x* and *y* A1: Any correct equation Must have *x* and *y* now.	M1A1
	Alternative – M1: identifies perpendicular bisector of (0, 0) and (1, -1) A1: $y = x - 1$		
	$(x-1)^2 + (x-1)^2 = 1$ or $y^2 + y^2 = 1$	Attempt to solve simultaneously tie obtain an equation in one variable and get to $x = \ldots$ or $y = \ldots$	M1
	$x = 1+\frac{1}{\sqrt{2}}, 1-\frac{1}{\sqrt{2}}$ or $y = \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$	Both (oe)	A1
	$\left(1+\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(1-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$	Both (oe) Pairs must be clearly identifiable but coordinate brackets not needed.	A1
			(7)
			Total 7

Question Number	Scheme		Marks
8.	$x = e^t$		
(a)	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$	Attempt to use an appropriate version of the chain rule	M1
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{1}{e^t} \left(= \frac{1}{x}\frac{dy}{dt} \right)$	Oe	A1
	$\frac{d^2y}{dx^2} = -\frac{1}{x^2}\frac{dy}{dt}, +\frac{1}{x}\frac{d^2y}{dt^2}\frac{dt}{dx}$ or $\frac{d^2y}{dx^2} = -\frac{1}{x^2}\frac{dy}{dt}, +\frac{1}{x^2}\frac{d^2y}{dt^2}$	M1: Use of the product rule (penalise chain rule errors by loss of A mark or marks) $\left(\text{Note } \ t = \ln x \Rightarrow \frac{dt}{dx} = \frac{1}{x}\right)$	M1 A1, A1
	$x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 13y = 0$ $\Rightarrow x^2.\frac{1}{x^2}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) + 5x\frac{1}{x}\frac{dy}{dt} + 13y = 0$ $\Rightarrow \frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 13y = 0*$	M1: Substitutes their first and second derivatives into the given differential equation Depends on both M marks above A1: Correct completion to printed answer	ddM1A1
			(7)
(b)	$m^2 + 4m + 13 = 0$ $\Rightarrow (m =)\frac{-4 \pm \sqrt{16-52}}{2}$	Attempt to solve the auxiliary equation	M1
	$(m =) -2 \pm 3i$	Correct roots May be implied by a correct GS	A1
	$y = e^{-2t}(A\cos 3t + B\sin 3t)$ or $y = Ae^{(-2+3i)t} + Be^{(-2-3i)t}$	Correct GS	A1
	$t = \ln x$		B1
	$y = \frac{A\cos(3\ln x) + B\sin(3\ln x)}{x^2}$ or $y = Ae^{(-2+3i)\ln x} + Be^{(-2-3i)\ln x}$		A1
			(5)
			Total 12

Question Number	Scheme		Marks
9.			
(a)	$a = 2a\sin 2\theta \Rightarrow \sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = ...$	$C_1 = C_2$ and attempt to solve for 2θ	M1
	$\sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$	$2\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ or both Decimals allowed (min 3 sf).	A1
	$\left(a, \frac{\pi}{12}\right), \left(a, \frac{5\pi}{12}\right)$	Both points Can be written $r = a$, $\theta = \frac{\pi}{12}, \frac{5\pi}{12}$ Decimals allowed (min 3 sf).	A1
			(3)
(b)	$\frac{1}{2} \times a^2 \times \frac{\pi}{3}$ oe	Correct expression for the sector	B1
	$\frac{1}{2}\int r^2 \mathrm{d}\theta = \frac{1}{2}\int \left(2a\sin 2\theta\right)^2 \mathrm{d}\theta$	Use of correct formula Limits not needed (ignore any shown)	M1
	$\cos 4\theta = 1 - 2\sin^2 2\theta$ $\Rightarrow \sin^2 2\theta = \frac{1}{2}\left(1 - \cos 4\theta\right)$	Uses $\sin^2 2\theta = \frac{\pm 1 \pm \cos 4\theta}{2}$	M1
	$\int \left(1 - \cos 4\theta\right) \mathrm{d}\theta = \theta - \frac{1}{4}\sin 4\theta$	Correct integration Limits not needed (ignore any shown)	A1
	$I = a^2\left[\theta - \frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{12}}$ $= a^2\left\{\left(\frac{\pi}{12} - \frac{1}{4}\sin 4.\frac{\pi}{12}\right) - (0)\right\}$	An attempt to find one or both of the regions either side of the sector. ie uses limits $0, \frac{\pi}{12}$ and/or $\frac{5\pi}{12}, \frac{\pi}{2}$, limits to be substituted and subtracted (if non-zero after substitution). Limits to be used the correct way round. If two integrals seen award mark if either correct. Both previous method marks must have been scored.	ddM1
	$R = 2I + \frac{a^2\pi}{6} = 2a^2\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) + \frac{a^2\pi}{6}$	Correct strategy for the complete area (sector + 2*I*). All areas must be positive.	M1
	$R = \frac{1}{12}a^2\left(4\pi - 3\sqrt{3}\right)$	If decimals seen anywhere (either in rt 3 or the limits) this mark is lost.	A1
			(7)
			Total 10

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